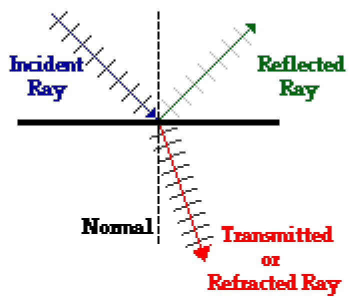
Unit 1: Waves
Lesson: Lenses and Refraction

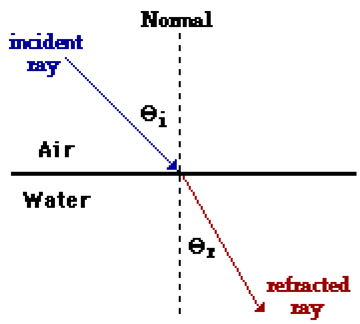
At a boundary, waves are partially reflected, partially transmitted
Less physically dense → More physically dense, v decreases, λ decreases

More physically dense→ Less physically dense, v increase, λ increases

Light behaves same way.



Air to water –



Refraction: bending of the path of a wave as it passes from one medium to another - caused by the change in speed of wave as it changes medium - amount of refraction (bending) depends on the angle of incidence.

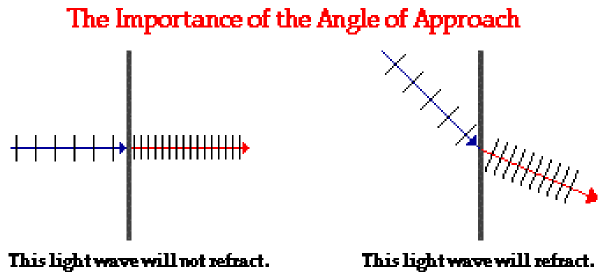
Conditions for refraction of light:

1. light wave must change speed when crossing boundary

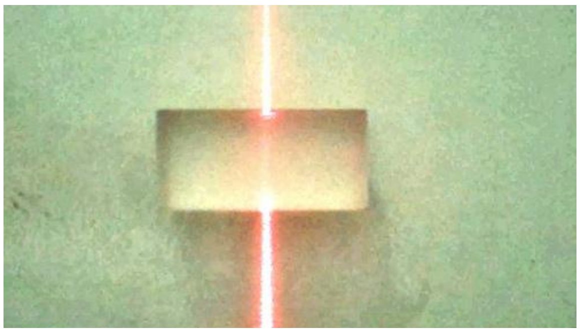
Less dense → More dense, v decreases, λ decreases

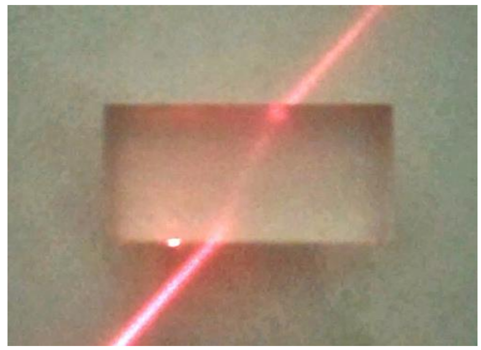
More dense→ Less dense, v increase, λ increases

2. light wave must hit the boundary at angle



As θ_i gets smaller, (the closer the incident ray is to the normal) less refraction occurs. When $\theta_i = 0$, **no refraction occurs.**





We can predict how light rays will bend.

Less dense → More dense (Fast medium → Slow medium) Ray will bend towards normal

FST

More dense → Less dense (Slow medium → Fast medium) Ray will bend away from normal

SFA

Optical Density & Light Speed:

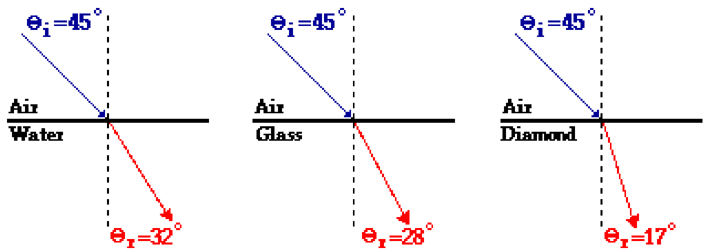
In vacuum, speed of light $c = 3 \times 10^8$ m/s
In matter, speed of light is less than c.
Optical density: sluggishness of atoms in a material to retransmit absorbed EM radiation.
More sluggishness, higher optical density

Index of Refraction (n): Indicator of Optical Density - measure of how much refraction light will undergo when moving from *vacuum* to a denser medium. (Vacuum - least optical density)

$$n_{material} = \frac{c}{v_{material}}$$

$$n_{material} \propto \frac{1}{v_{material}}$$

Examples:



Material	**Index of Refraction**	
Vacuum	**1.0000**	**<--lowest optical density**
Air	**1.0003**	
Ice	**1.31**	
Water	**1.333**	
Ethyl Alcohol	**1.36**	
Plexiglas	**1.51**	
Crown Glass	**1.52**	
Light Flint Glass	**1.58**	
Dense Flint Glass	**1.66**	
Zircon	**1.923**	
Diamond	**2.417**	
Rutile	**2.907**	
Gallium phosphide	**3.50**	**<--highest optical density**

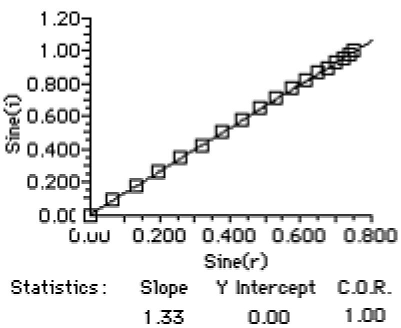
Snell's Law: Relationship between angles of incidence and refraction, and the indices of refraction of the 2 media.

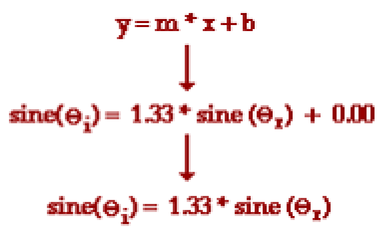
$$n_i sin\, \theta_i = n_r sin\theta_r$$

For a given frequency, and a pair of substances, the ratio $sin\, \theta_i : sin\theta_r$ is constant.

$$\frac{sin\, \theta_i}{sin\, \theta_r} = \frac{n_r}{n_i}$$

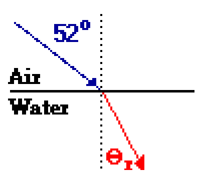
Imagine the angle of Incidence and Refraction were found for light coming from air to water. The angles themselves would have no apparent relationship. However, if the sine of the angle of incidence and the sine of the angle of refraction were plotted, the plot would be a straight line, indicating a linear relationship between the sines of the important angles.





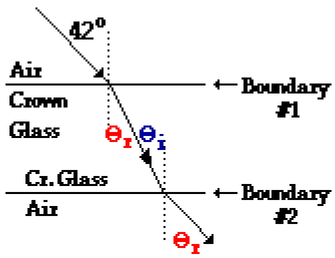
Example 1:

A ray of light in air is approaching the boundary with water at an angle of 52 degrees. Determine the angle of refraction of the light ray. (n $_{H2O}$ = 1.33)
(Ans: 36°)



Example 2: A ray of light in crown glass exits into air at an angle of 25°. Determine the angle at which the light approached the glass-air boundary. Refer to the table of indices of refraction if necessary. (Ans: 16°)

Example 3:
A ray of light in air is approaching a layer of crown glass at an angle of 42.0 degrees. Determine the angle of refraction of the light ray upon entering the crown glass and upon leaving the crown glass. (Ans: 26°, 42°)

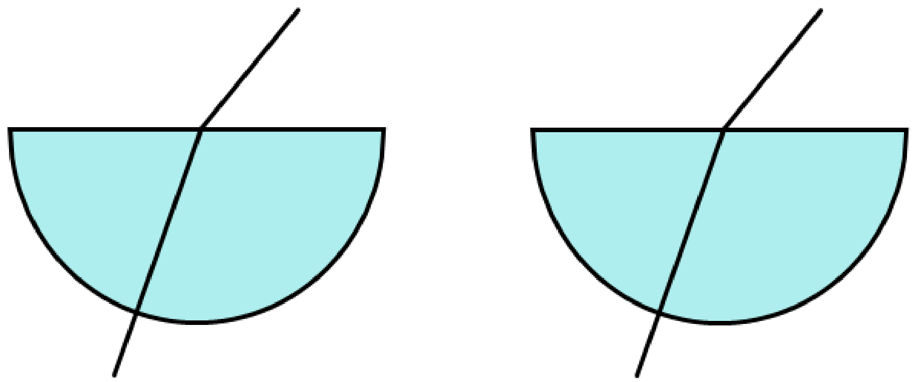


Important conceptual idea

When light approaches a layer that has the shape of a parallelogram that is bounded on both sides by the same material, then the angle at which the light enters the material is equal to the angle at which light exits the layer.

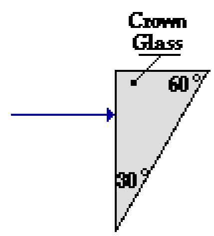
If the layer is not a parallelogram or is not bound on both sides by the same material, then this will not be the case.

Example 4:



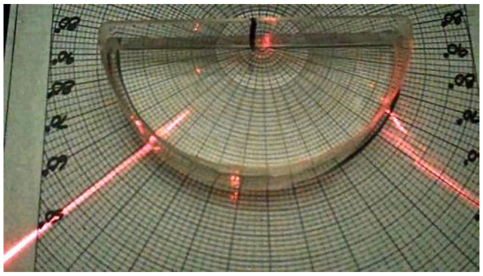
Example 5:

A ray of light in air is approaches a triangular piece of crown glass at an angle of 0.00 degrees (as shown in the diagram at the right). Find the angle at which the ray exits the crown glass. (Ans:49.5°)

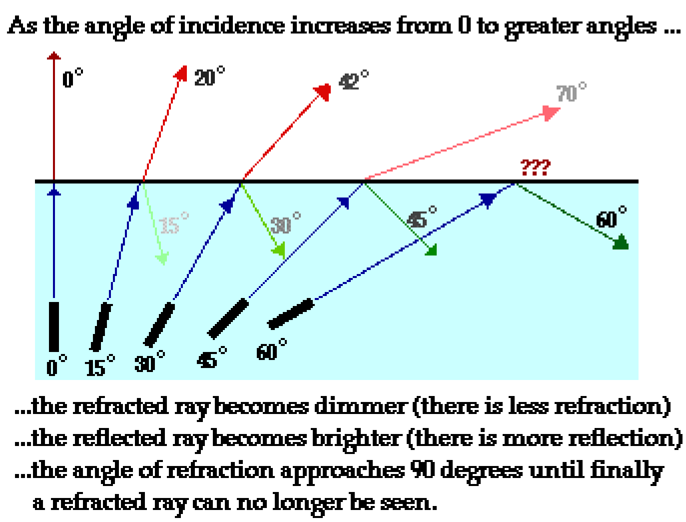


Critical angle: angle of incidence for which angle of refraction = 90°

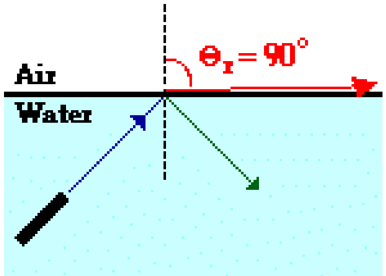
Total Internal Reflection – (TIR) involves the reflection of the entire incident light off the boundary. TIR only takes place when both of the following two conditions are met:



- light **must go** from more optically dense → less optically dense
- angle of incidence is greater than the **critical angle.**

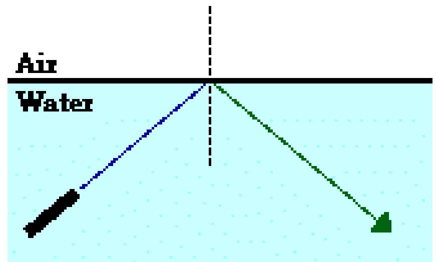


Reflection and Refraction



When the angle of incidence equal the critical angle, the angle of refraction is 90-degrees.

Total Internal Reflection



When the angle of incidence is greater than the critical angle, all the light undergoes reflection.

To calculate critical angle:

$$n_i \sin\theta_i = n_r \sin\theta_r$$

$$n_i \sin\theta_c = n_r \sin 90°$$

and since $n_r = 1$ for air

$$\sin\theta_c = \frac{1}{n}$$

Example 1:
Calculate the critical angle for the crown glass-air boundary. (Ans: 41.1°)

Example 2:
Calculate the critical angle for the diamond-air boundary. (Ans: 24.4°)

Applications/ occurrences of TIR:

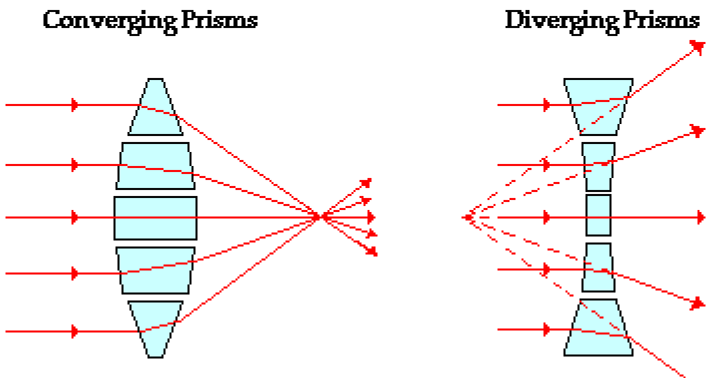
Fiber optics, cut of a diamond, some periscopes, mirages

Applications/ occurrences of refraction”
Dispersion through a prism, rainbows, mirages

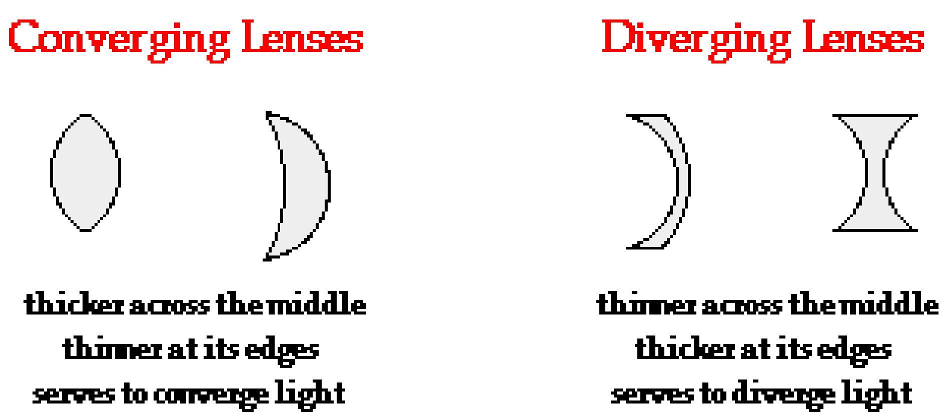
LENSES

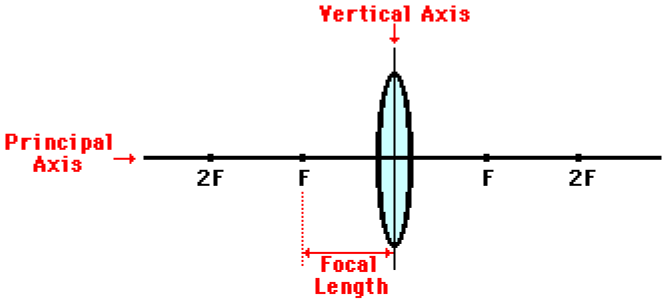
The Anatomy of a Lens

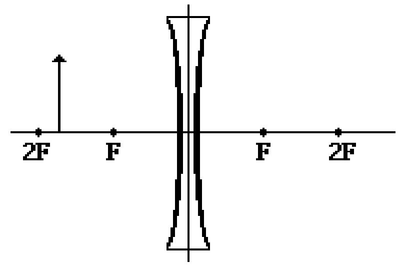
Lens: piece of transparent material - refracts light rays in such a way as to form an image - can be thought of as a series of tiny refracting prisms.When these prisms act together, they produce a bright image focused at a point.



A set of prisms acting as a converging and diverging lens.







Vertical axis: line that bisects the symmetrical lens into halves.

Focal point: A lens has two possible focal points. Light rays incident towards either face of the lens and traveling parallel to the principal axis will either converge (converging lens) to a focal point, or appear to diverge (diverging lens) from a focal point.

The focal point is denoted by the letter **F** on the diagrams below.

Principal axis: horizontal line passing through the center of the lens along which lie the points 2F and F.

Image Location Rules for a Converging Lens

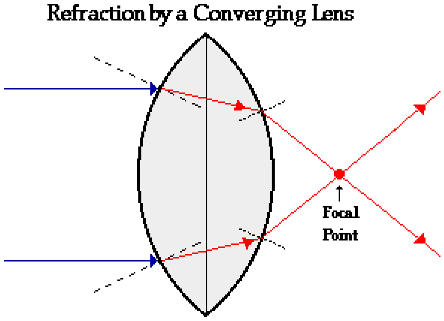
1. IR parallel to principal axis, refracts through lens and travels through the far F.

2. IR through near F on the way to the lens, refracts through lens and comes out parallel to the principal axis.

3. IR passing through the center of the lens continues unchanged.

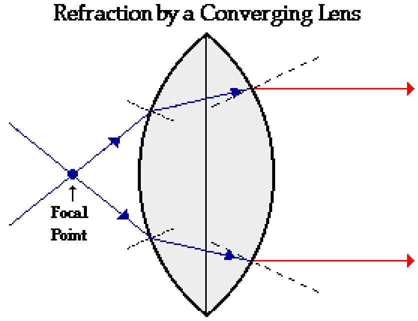
4. Intersection of the RRs is the image location. (When required, extend both RRs backwards through the lens until the RRs intersect).

Refraction by a Converging Lens



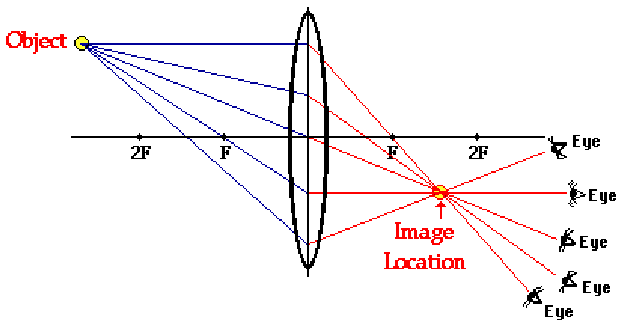
Incident rays which travel parallel to the principal axis will refract through the lens and converge to a point.

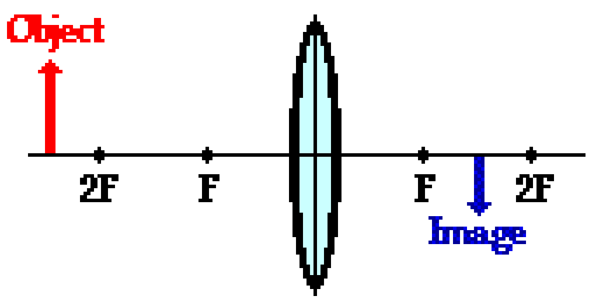
Refraction by a Converging Lens

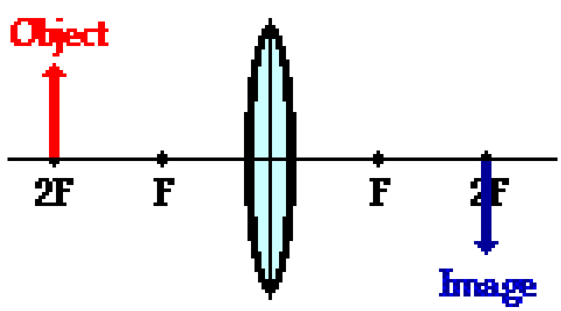


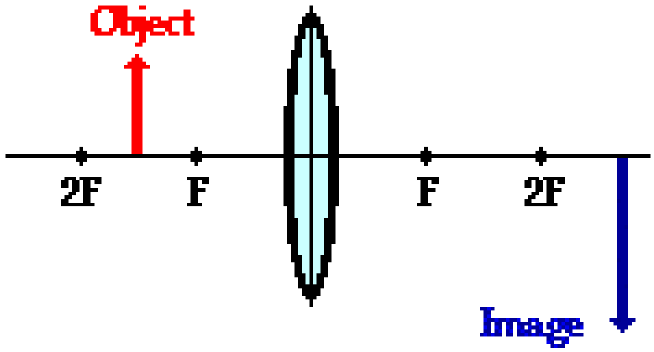
Incident rays which travel through the focal point will refract through the lens and travel parallel to the principal axis.

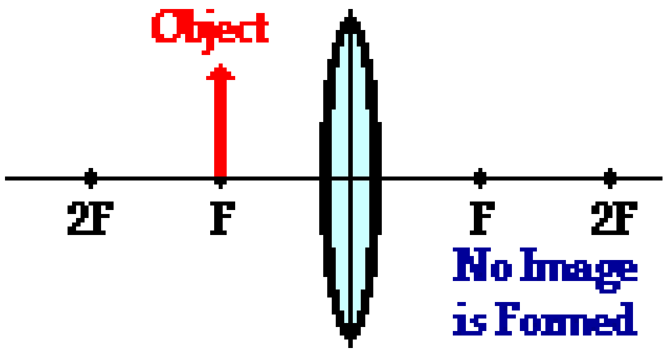
Image Formation by a Converging Lens








Object
2F
F
F
2F
Image


Object
2F
F
F
2F
No Image
is Formed

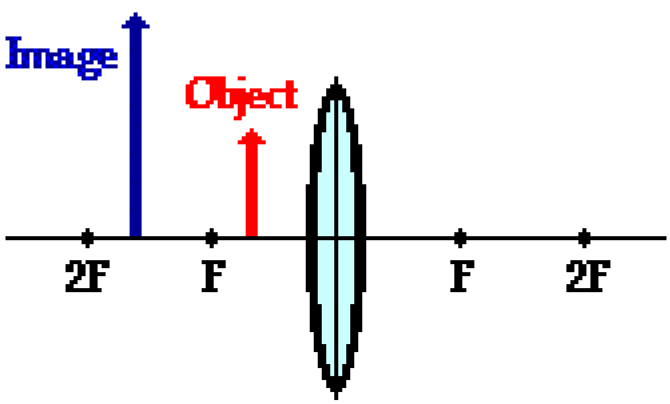


Image
Object
2F
F
F
2F

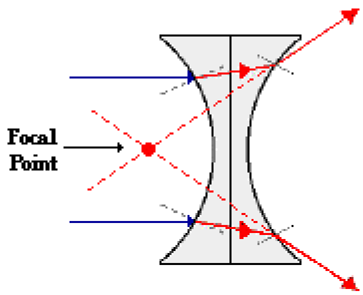
Image Location Rules for a Diverging Lens

1. IR parallel to the principal axis, refracts through the lens and travels *in line with* the near F (i.e., in a direction such that its backward extension will pass through near F).

2. IR traveling towards the far F on the way to the lens refracts through the lens and travels parallel to the principal axis.

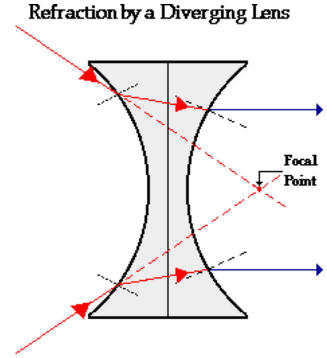
3. IR passing through the center of the lens will in effect continue unchanged.

4. Extend both refracted rays backwards through the lens until the RRs intersect. The intersection of the RRs is the location of the image.



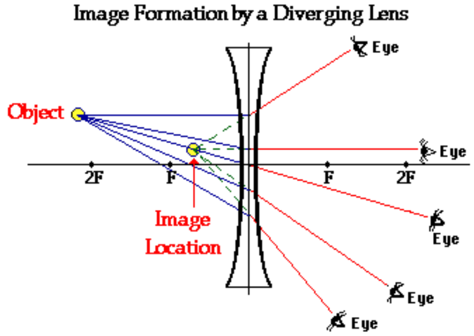
A diverging lens is said to have a negative focal length since rays which enter the lens traveling parallel to the principal axis diverge.

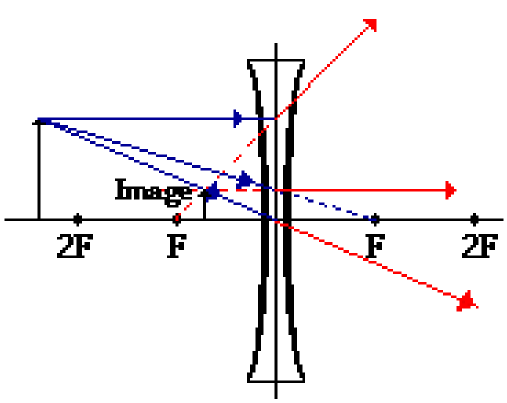
Refraction by a Diverging Lens

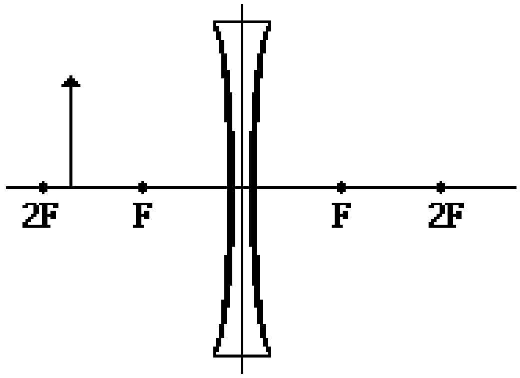


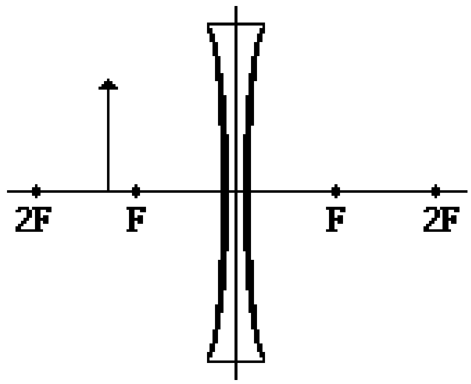
Incident rays traveling towards the focal point will refract and travel parallel to the principal axis.

Image Formation by a Diverging Lens









The Mathematics of Lenses

Prediction of image location:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Image height:

$$h_i = -\frac{h_o d_i}{d_o}$$

Magnification ratio:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Sign Conventions

- f is + if the lens is a double convex lens (converging lens)
- f is - if the lens is a double concave lens (diverging lens)
- d_i is + if the image is a real image and on the opposite side of the lens.
- d_i is - if the image is a virtual image and on the object's side of the lens.
- h_i is + if the image is an upright image (and therefore, also virtual)
- h_i is - if the image an inverted image (and therefore, also real)

Example 1:

A 4.00-cm tall light bulb is placed a distance of 45.7 cm from a double convex lens having a focal length of 15.2 cm. Determine the image distance and the image size.
(Ans: d_i = 22.8 cm, h_i = -1.99 cm)

Example 2:

A 4.00-cm tall light bulb is placed a distance of 8.30 cm from a double convex lens having a focal length of 15.2 cm. (NOTE: this is the same object and the same lens, only this time the object is placed closer to the lens.) Determine the image distance and the image size. (Ans: **d_i = -18.3 cm, h_i = 8.81 cm)**

Example 3:

A 4.00-cm tall light bulb is placed a distance of 35.5 cm from a diverging lens having a focal length of -12.2 cm. Determine the image distance and the image size.
(Ans: d_i = -9.08 cm, h_i = 1.02 cm)

Lens Combinations (Honors)

The final image produced by a compound lens system can be found by treating the image of one lens as the object for the adjacent lens.

a. If the image for the first lens (L1) is formed in front of the second lens (L2), the object for the second lens is said to be real.
b. If the rays pass through the second lens before the image is formed from the first lens – the object for the second lens is said to be virtual and the object distance for the second lens is taken to be negative when calculating the image distance.

To calculate the final magnification of the image: